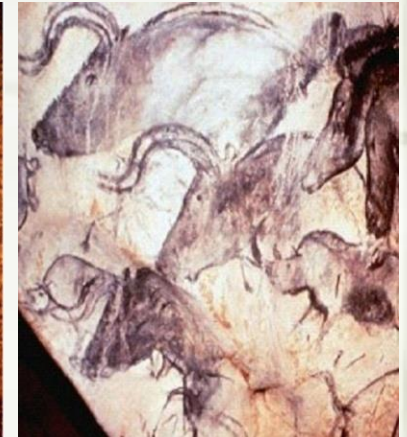
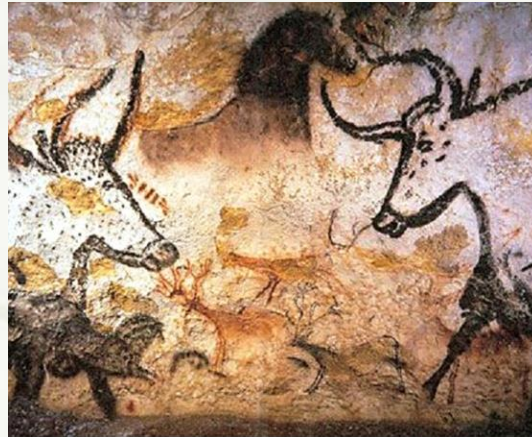


Morphological track

Shapes

- Painting of living beings on cave walls at Lascaux [about 1500th BC]

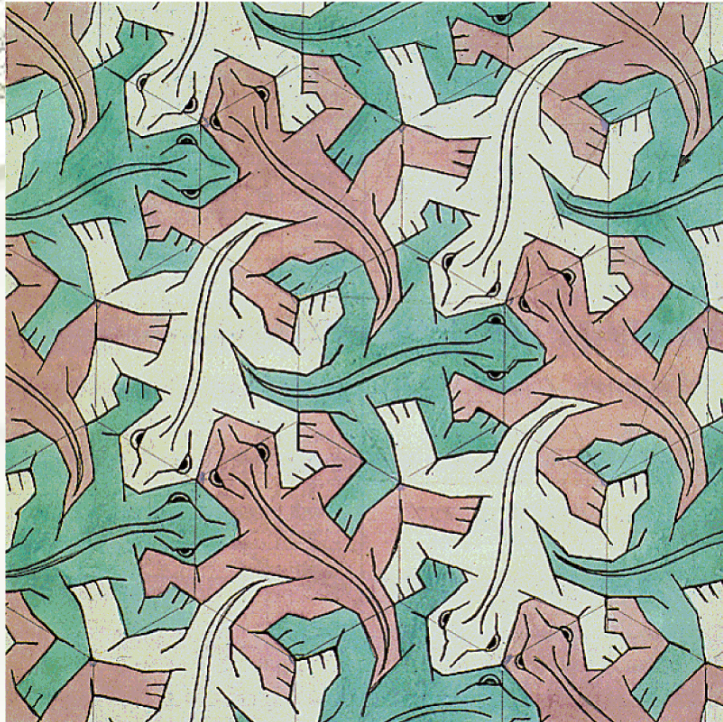


- L'homme qui marche by Alberto Giacometti, 1948, NOUVELLES IMAGES Editor (1976)
- Les lutteurs by Honoré Daumier, 1852, Lyon, Musée des Beaux-Arts
- Nu bleu II by Henri Matisse, 1952, Collections du Musée National d'Art Moderne, Centre Pompidou, Paris.



Tessellation

- ★ A **tessellation** or a tiling is a way to cover the plane with shapes so that there is no overlapping or gaps.



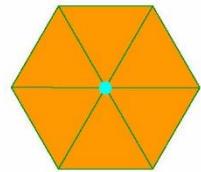
Study of Regular Division of the Plane
with Reptiles by Maurits C. Escher.

Regular Tessellation

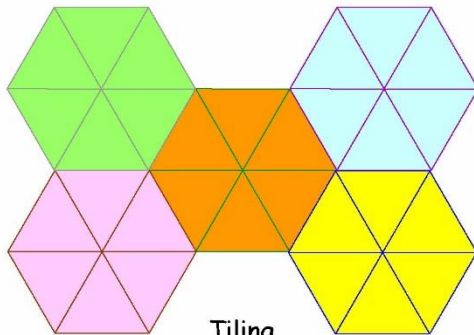
- The tiles must be copies of one regular polygon.
- Each vertex must join another vertex.

Regular tiling

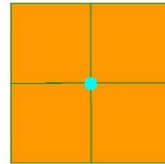
- So, the only regular polygons that tessellate the plane are triangles, squares and hexagons.



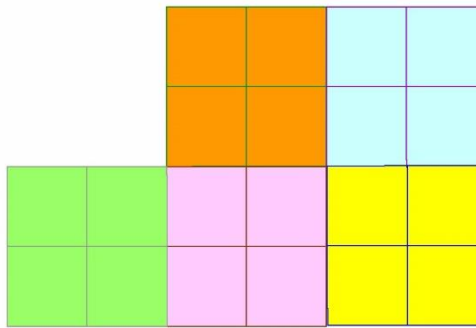
Base Tile: 3.3.3.3.3.3



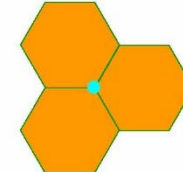
Tiling



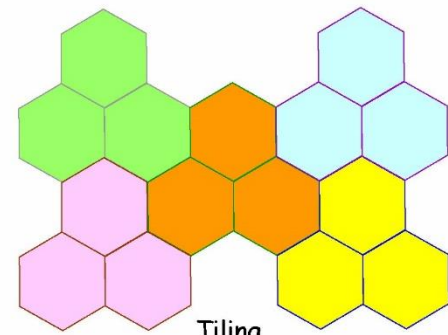
Base Tile: 4.4.4.4



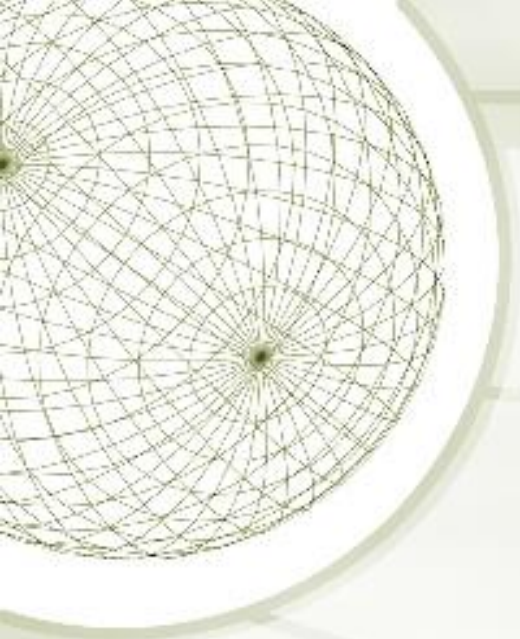
Tiling



Base Tile: 6.6.6



Tiling

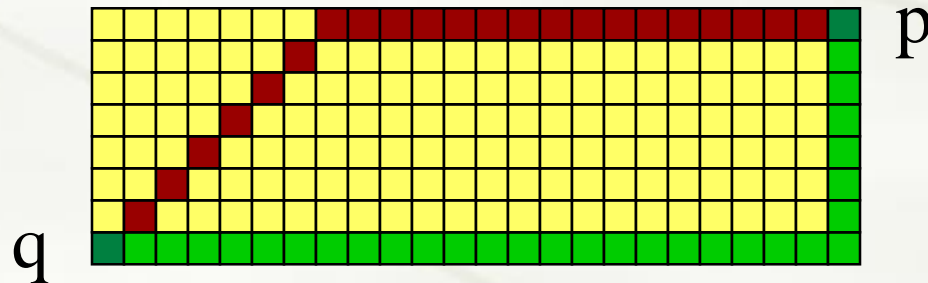


Contour based representations

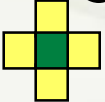
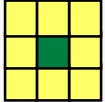
Preliminary statements: binary images

- ★ Let $F=\{1\}$ et $F^*=\{0\}$ be the two sets constituting a binary image on a **square** plane tessellation. Let call them **foreground** (image) and **background** respectively
- ★ Given two pixels p and q , having coordinates (i,j) and (h,k) , the two distance functions **d_4 (called city block)** et **d_8 (chessboard)** are defined as:

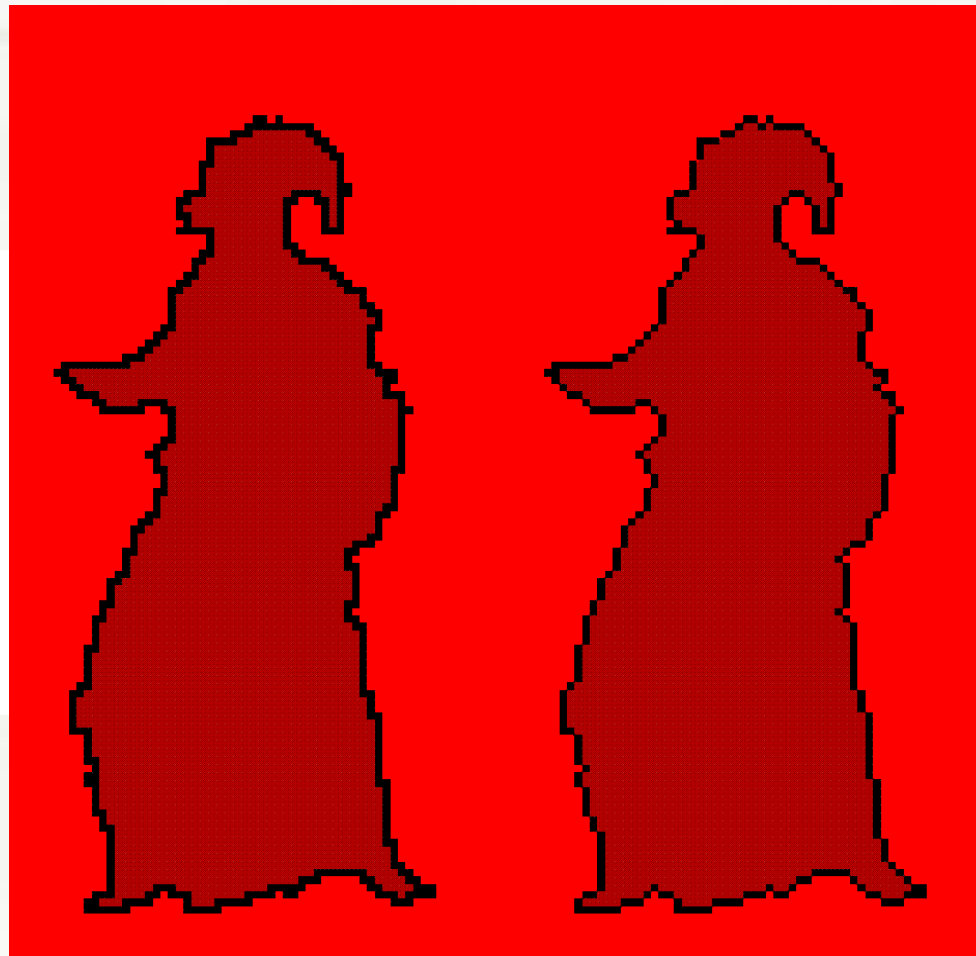
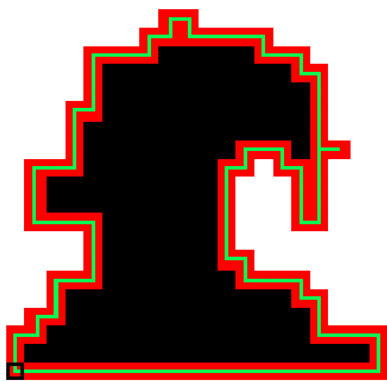
$$d_4(p,q) = |i-h| + |j-k| \quad d_8(p,q) = \max \{|i-h|, |j-k|\}$$



Preliminary statements

- ★ The neighbors of pixel p are all pixels that have unitary distance from p
- ★ The set of neighbors constitutes the neighborhood of p . Depending on the adopted square metric we consider the $N_4(p)$  or $N_8(p)$  neighborhoods.
- ★ A **path** of length n from p to q is a sequence of pixels $p=p_0, p_1, p_2, \dots, p_n=q$ that in the adopted metric has p_i neighbor of p_{i+1} , $1 \leq i \leq n$
- ★ A subset of F (or of F^*) is **connected** if for each couple of pixels in the subset exist a path between them entirely belonging to the subset.
- ★ The **contour** C of an image F is the subset of F having unitary distance from F^*

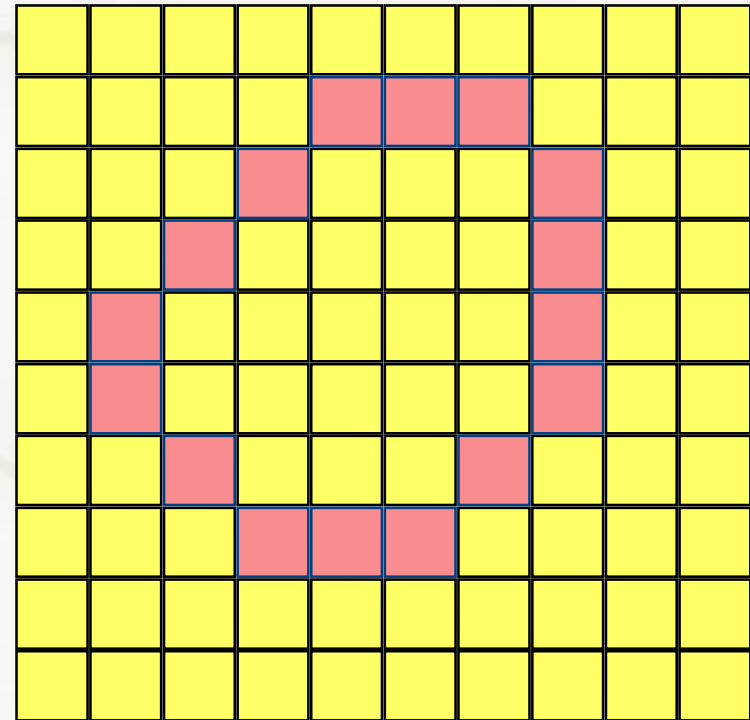
4 and 8 connectivity



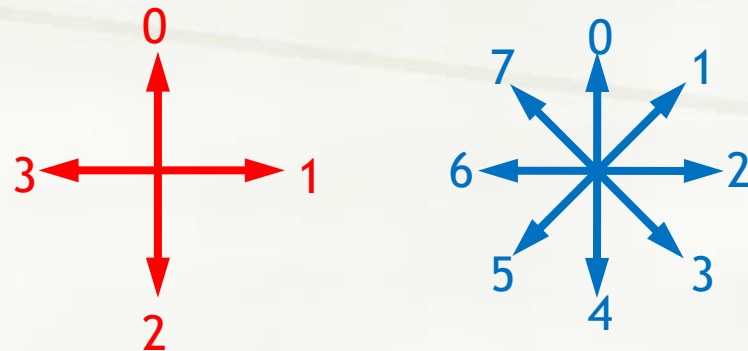
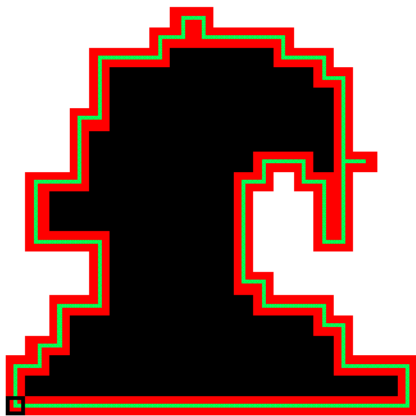


Topological Paradox

- ✦ Contour in 8 connectivity
- ✦ F and F* are separated only in 4 connectivity



The Herbert Freeman code



4-contour having 102 codes

001010011000333000110001000111010
121111211212222132222300030332322
22212111212211122333333333333333
333

8-contour having 81 codes

001102107660002100100221123222323
444354446007755444332234322446666
6666666666666666

The Freeman code: characteristics

- ★ The length of the contour is the number of codes n in N_4 and in N_8 is given by: $L_8 = n_{\text{even}} + \sqrt{2} n_{\text{odd}}$

- ★ The coordinates of code k are given by:

$$\begin{array}{l} Cx^1=Cx^2=Cx^3=Cy^7=Cy^0=Cy^1=1 \\ Cx^0=Cx^4=Cy^2=Cy^6=0 \\ Cx^5=Cx^6=Cx^7=Cy^3=Cy^4=Cy^5=-1 \end{array} \quad x^k = \sum_{i=0}^k c_x^i \quad y^k = \sum_{i=0}^k c_y^i$$

- ★ The area is given by:

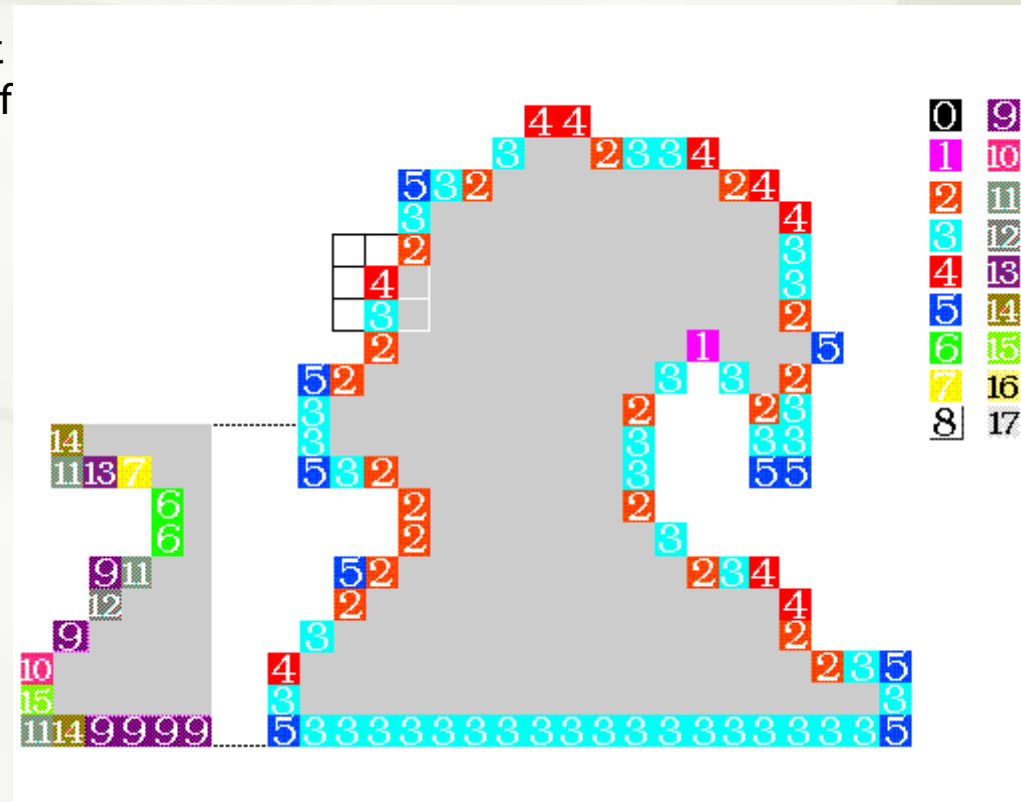
$$A = 1 + \sum_{i=0}^n c_x^i \sum_{j=0}^{i-1} c_y^j + \frac{1}{2} \sum_{i=0}^n |c_x^i| + |c_y^i|$$

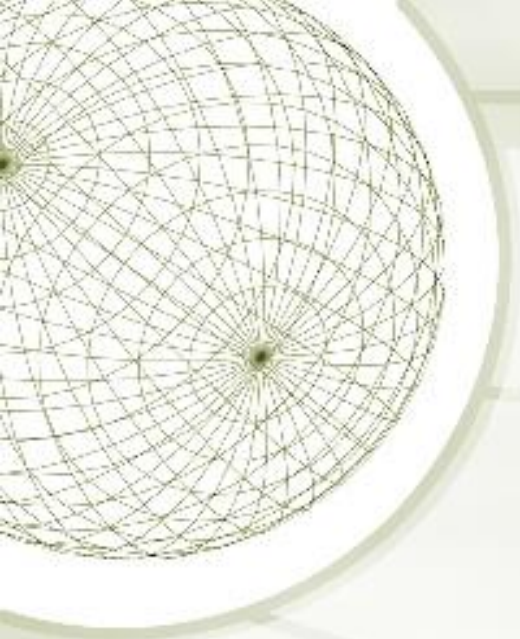
- ★ Closed contour conditions:

$$n_7+n_0+n_1=n_3+n_4+n_5 \quad n_5+n_6+n_7=n_1+n_2+n_3.$$

Concavities and convexities

- ★ A simple technique for extracting such properties, uses 3x3 windows iteratively. It is based on the evaluation of the number of background pixels in such windows
- ★ For the successive steps, the evaluation now corresponds to the addition of values of the contour labels within the sub-array (including the central pixel label)
- ★ The second iteration has only been applied to a fragment of the whole picture, and models a wider receptive field equivalent to a 5x5 window.
- ★ If this process is continued, the higher the number of iterations, the larger the receptive field, the lower the spatial quantization effects, so obtaining a more comprehensive description, although at a lower resolution.

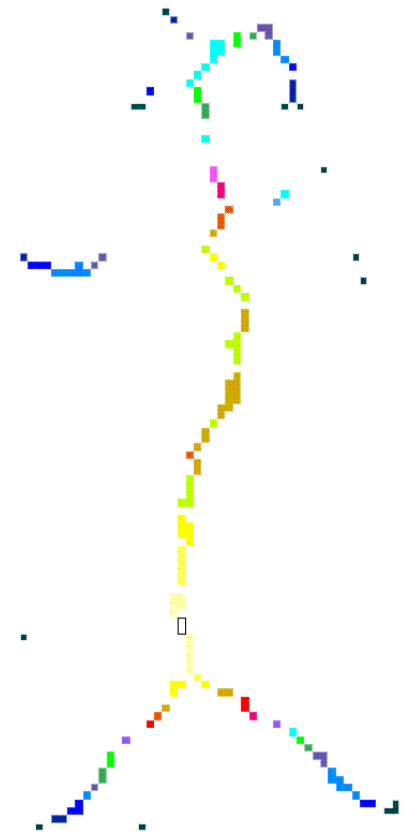
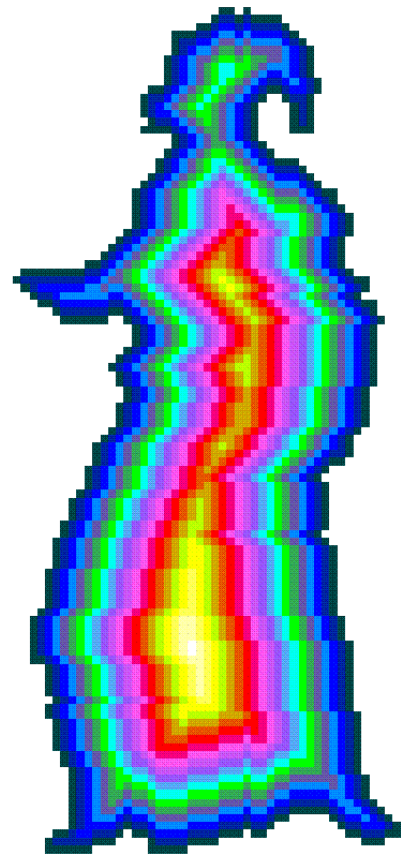




Region based representations

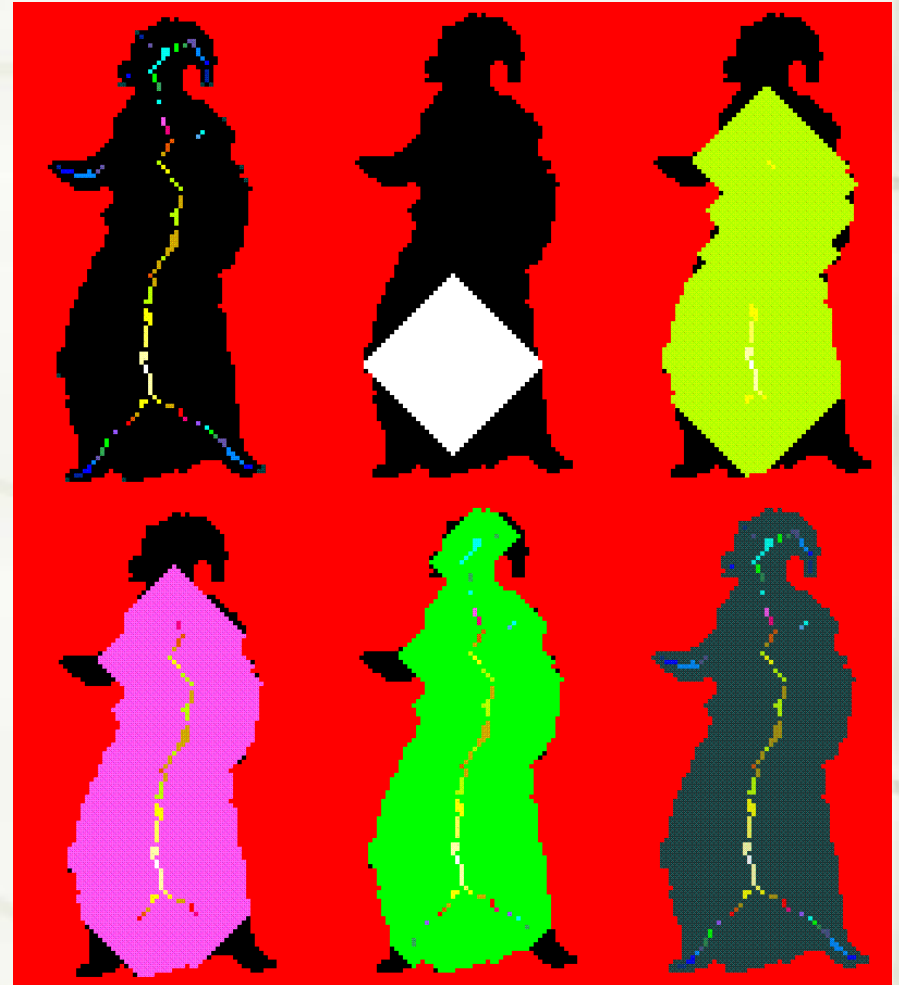
Distance transform and MAT

- ✦ The **Distance Transform (DT)** is obtained by labeling all the pixels inside a binary object with their distance to the background
- ✦ Applying twenty iterations of the erosion operator (structural element: unit disk) twenty successive colored layers showing equi-distant contours from the background for a Manhattan distance metric are obtained
- ✦ Every pixel has a color corresponding to its distance label which increases going inwards. In practice, this value represents the side of the greatest digital disk having its centre on this pixel, which is completely contained in the binary object.
- ✦ Any pattern can be interpreted as the union of all **its maximal digital disks**. A maximal disk is a disk contained in the object that is not completely overlapped by any other disk.
- ✦ The set of the centers of the maximal disks with their labels, constitutes the **MAT**



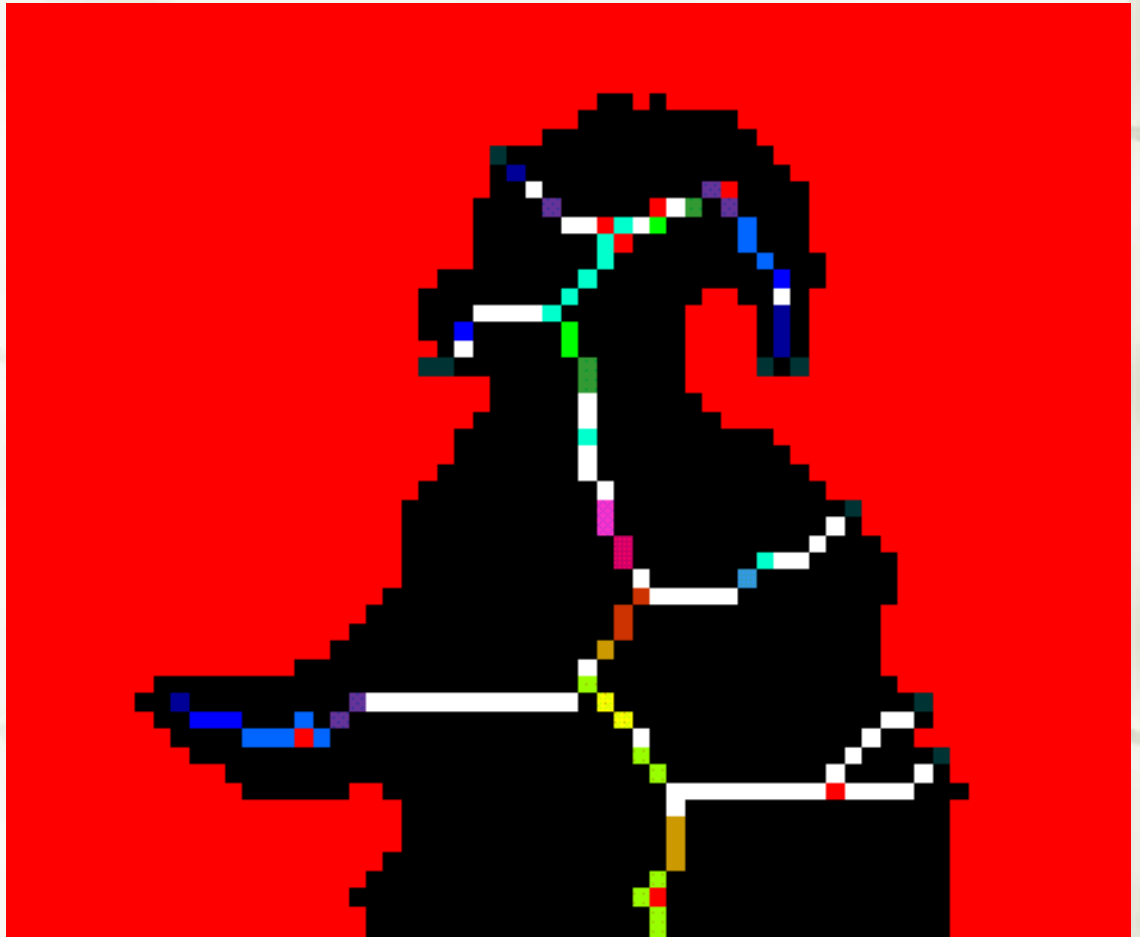
Reverting progressively MAT

- ✦ A procedure to derive the MAT from the DT is based on the comparison of neighboring labels to establish whether a local maximum exists
- ✦ This transform is complete in the sense that it is possible to revert it, so obtaining the original object back
- ✦ This recovery process can be implemented by expanding every pixel belonging to the MAT, using the corresponding maximal disc whose size is given by the pixel label. The logical union of all such discs reconstructs the original object
- ✦ This figure shows the progressive reconstruction, starting from the set of disks corresponding to the highest level (two white disks) until the sixth and last monk's profile, where discs, reduced to just one pixel, have been included
- ✦ This transform is compact since the full object may be described only by its labeled disk centers



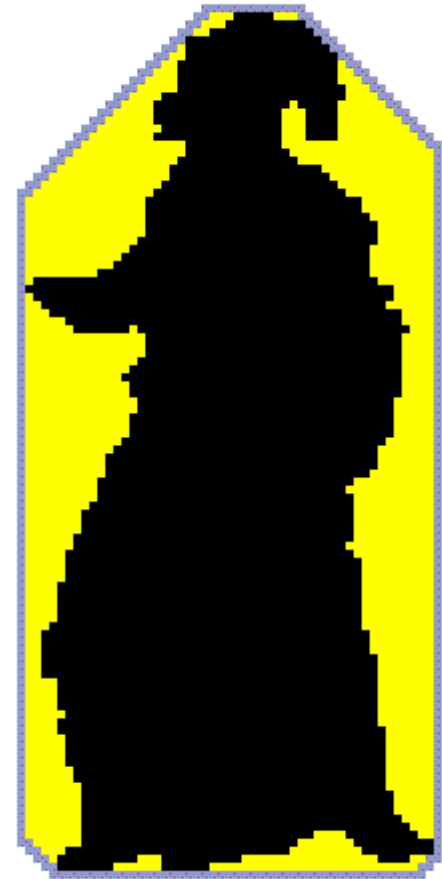
Skeleton from MAT

- ★ MAT does not ensure connectivity for a connected object
- ★ Many different algorithms were designed for generating skeletons. *A simple one proceeds in two steps:*
 - ★ *find gaps between different branches and bridge them by joining the extremes along paths with directions dependent on context (white pixels);*
 - ★ *thin the obtained branches so as to produce a one-pixel wide representation (red pixels)*
- ★ One possible application of the skeleton is contour smoothing, where pruning branches with relatively short lengths and then inverting the skeleton transformation, a more regular contour is obtained
- ★ The skeleton transformation is not reversible in general



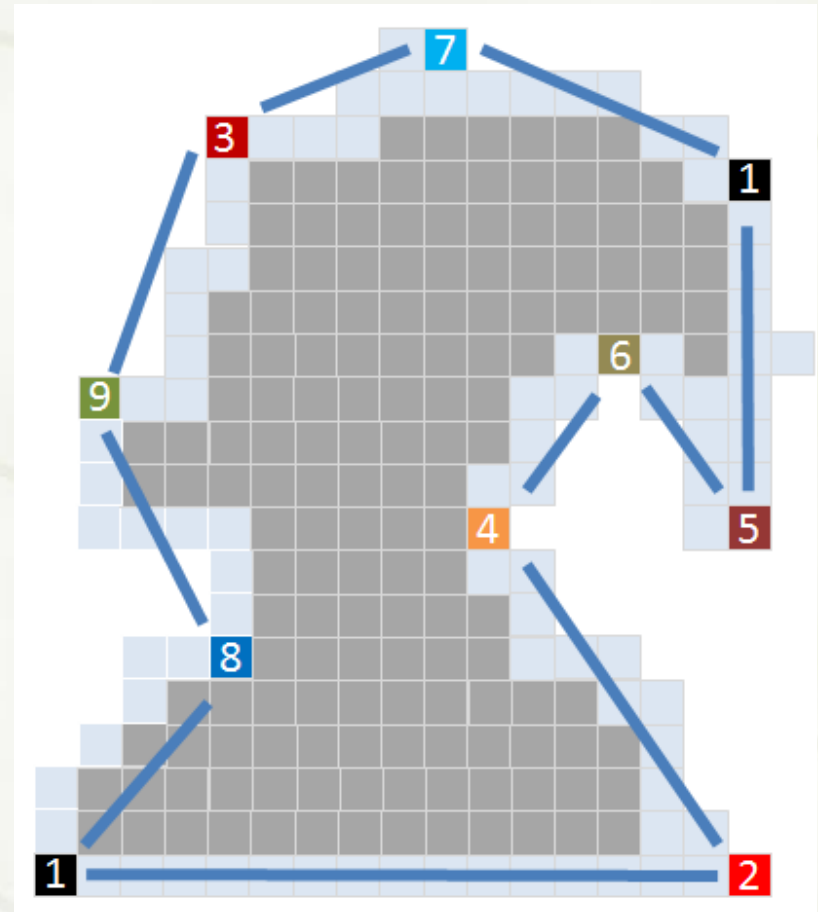
8-Convex Hull

- ★ The *convex hull* is the minimum n -sided convex polygon that completely circumscribes an object, gives another possible description of a binary object. An example is given in figure where a constrained 8-sided polygon has been chosen to coarsely describe the monk silhouette.
- ★ To obtain the convex hull a simple algorithm propagates the object along the eight (more generally $2n$) orientations and then: i) logically OR the opposite propagated segments; and ii) logically AND the four (more generally n) resulting segments. The contour of the obtained polygon is the convex hull.



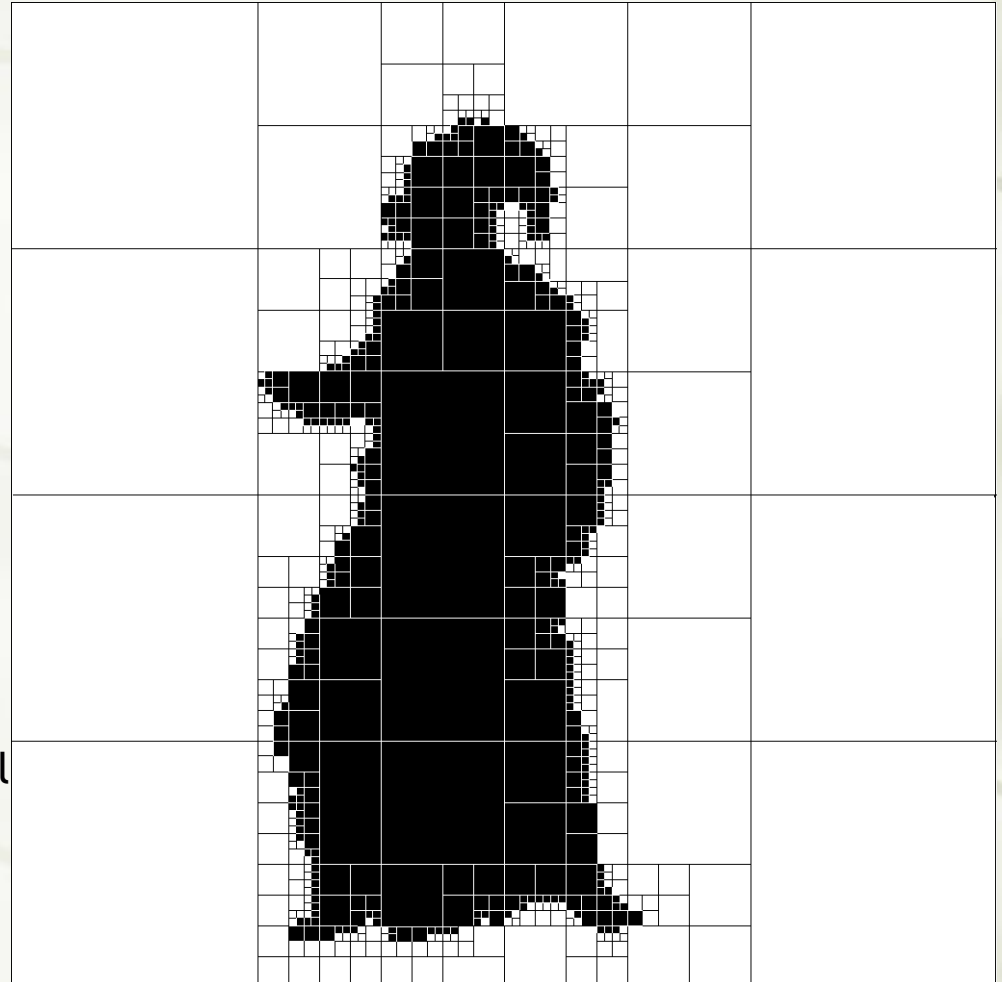
Polygonal approximations of contour

- ✦ An algorithm schema for contour polygonal approximation is:
 - ✦ find a diameter d of a closed digital line, or consider the extremity-linking segment els of a non-closed digital line;
 - ✦ find the furthest pixel from d or els to the line, and draw both segments joining this pixel to d or els extremities;
 - ✦ for all segments s , find the point p_s of the digital line with maximum distance δ_s to s .
 - ✦ select the maximum distance δ_{s^*} over all segments and draw the two new segments joining the corresponding p_{s^*} to extremities of the related segment s^* ;
 - ✦ stop when δ_{s^*} is below a preset threshold or at a preset number of segments
- ✦ The approximation at step four of the monk's head contour: construction follows the order red, orange, green and blue; the starting diameter extremes are black pixels.



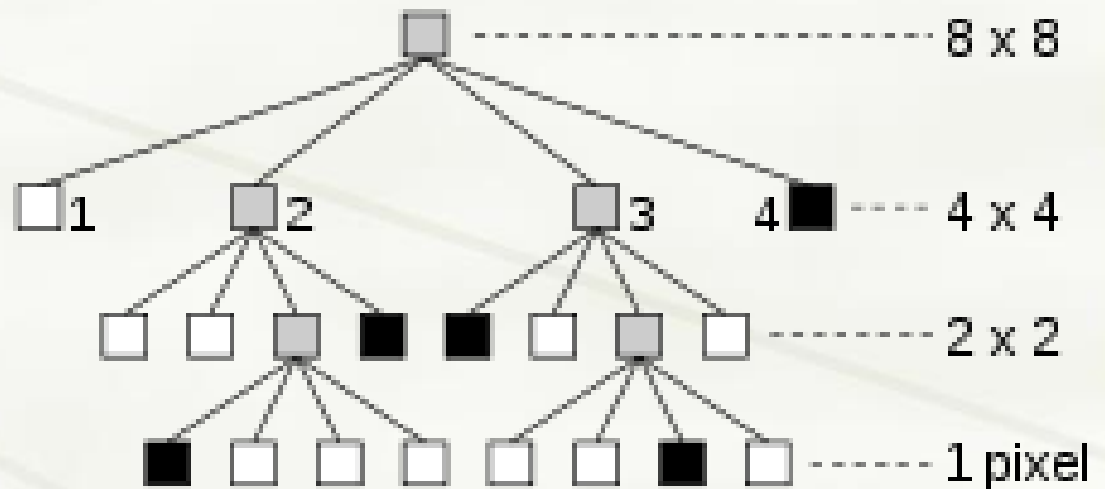
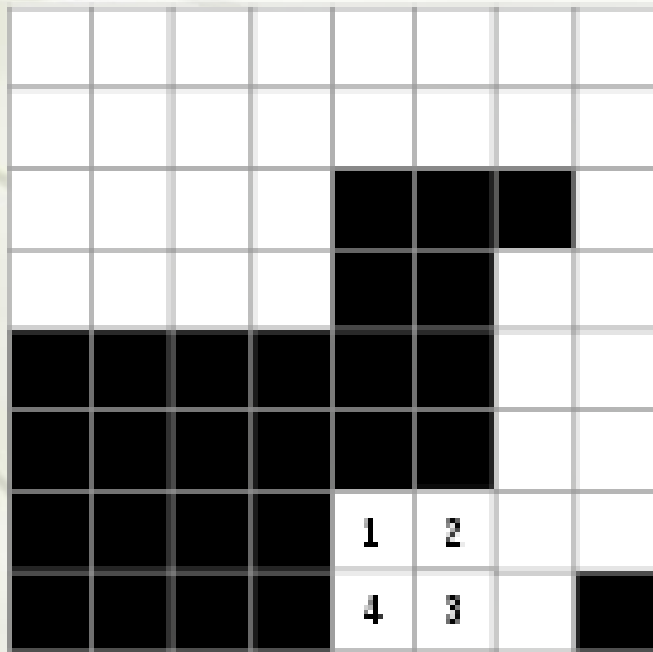
The quadtree representation

- ✦ A quadtree construction process:
 - ✦ Starting from the full image a four square splitting process is performed, and the subdivision process is then applied recursively. This process stops whenever reaching a homogeneous square (background/foreground)
 - ✦ The homogeneous squares are single colored (0/1 respectively for background/foreground) while others require further subdivisions in order to cater for finer detail. The process ends when no more mixed squares exist
- ✦ The transformation is totally reversible
- ✦ Due to the privileged orientation, the obtained blocks may not be the maximal ones included in the object. Moreover, an object translation may result in a different decomposition.



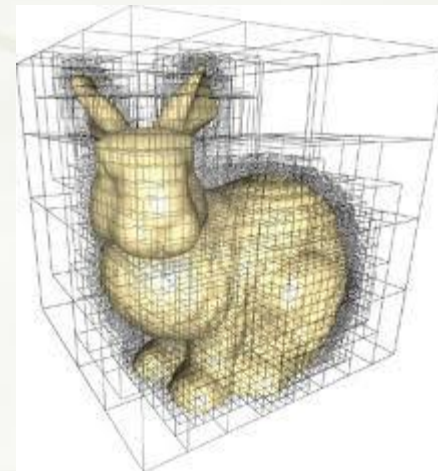
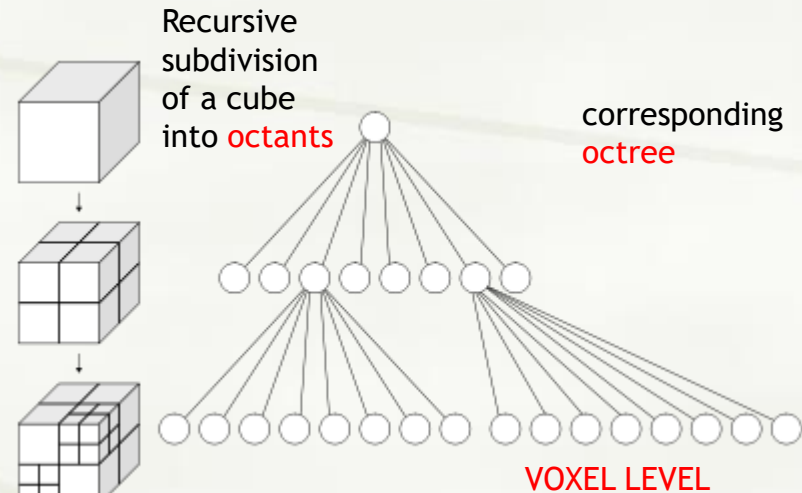
The quadtree representation

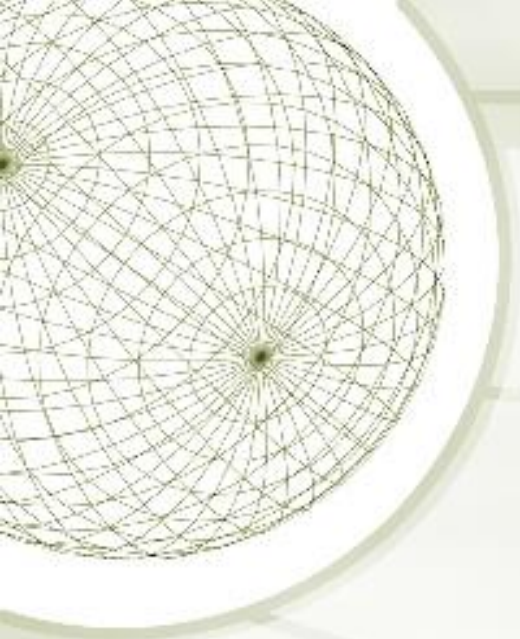
- ✦ The quadtree is an effective **hierarchical** data structure. The hierarchy allows, selecting an adequate resolution level for the task at hand, the implementation of a version of the focus of attention on salient details of the image



The octree representation

- ★ An **octree** is a **tree data structure** in which each internal node has exactly eight children.
- ★ Octrees are most often used to partition a three dimensional space by recursively subdividing it into eight octants.
- ★ Octrees are the three-dimensional analog of quadtrees.
- ★ Octrees are often used in 3D graphics and 3D game engine

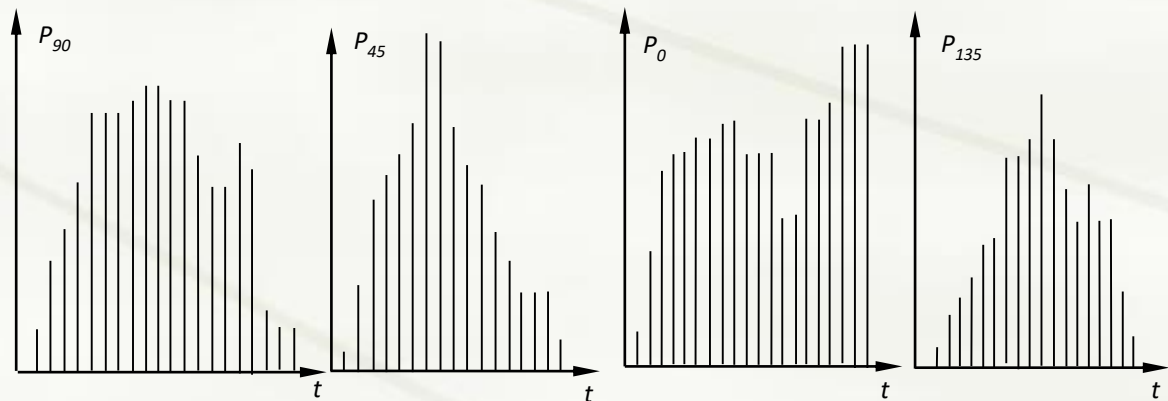
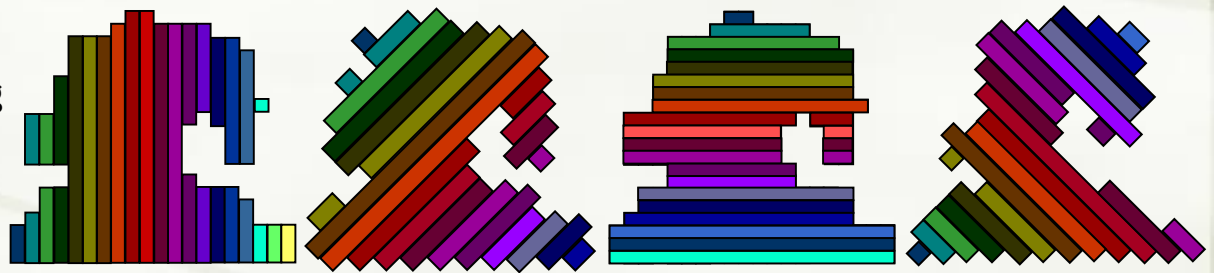




Shape representation by projections

Shape representation by projections

- ✦ The shape of a binary region is represented by a collection of segments each one representing its length in a given direction θ and at a given position t
- ✦ The thickness of a “quasi” binary pattern generalizes into the sum of grey levels along the corresponding ray. The sequence of such sums can be gathered into a function $P_\theta(t)$ called the projection of the pattern in the direction θ . The function of two variables $R(\theta, t)$ defined by the collection of the $P_\theta(t)$'s when θ varies from 0 to π is called the *Radon transform* of the pattern



Pattern recognition without reconstruction

- ★ Reconstruction is not always necessary for recognizing patterns in many practical cases. As an example, if there are only the three key patterns by using their vertical or horizontal projections (323, 322, 313 and 323, 313, 232 respectively) they can be discriminated.
- ★ This is not the case any more, even in the absence of noise, when two different characters whose vertical and horizontal projections are 232, 313 and 313, 322 respectively. Two new characters may be discriminated among the five ones by using both vertical and horizontal projections. The larger the character set, the harder! Ambiguities grow with the number of characters requiring more projections to fight symmetries, as in the first couple or partial translations as in the second couple.
- ★ Projections represent a quick and easy coding system that can be exploited in many practical recognition cases as with QR-code matrices developed in Japan. Furthermore by using such projection both computation time and complexity can be strongly diminished quickly arriving to practical solutions.

